

Renormalons in exclusive meson electroproduction**A.V. Belitsky**

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Renormalons in exclusive meson electroproduction

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We discuss the possibility of measuring generalized parton distributions in exclusive electroproduction of mesons off the nucleon and estimate the uncertainty from perturbatively induced higher-twist corrections. We find that, while the magnitude of the cross section changes significantly taking into account twist-four contributions modeled via renormalons, the transverse spin asymmetry is weakly sensitive to them and displays the precocious scaling.

Generalized parton distributions (GPDs) $F(x, \xi, \Delta^2)$ encode exhaustive information on one-particle correlations in the nucleon and thus carry the lore on its wave function and the phase structure of the latter. The quantum-mechanical wave function Ψ allows to predict expectation values of all observable for a given system. An identical description is achieved by means of the density matrix $\rho(x_1, x_2) = \Psi^*(x_1)\Psi(x_2)$. The latter can be used in turn to construct the quantum equivalent of the classical phase-space distribution, the primary example being known as the Wigner quasi-probability function $W(k, r) = \int \frac{dx}{2\pi\hbar} e^{-ikx/\hbar} \rho(r - \frac{1}{2}x, r + \frac{1}{2}x)$. Contrary to its classical counterpart it is not positive definite, — a hallmark of the interference. The marginals of $W(k, r)$ acquire however the probability interpretation as coordinate density $\rho(r) = \int dk W(k, r) = |\Psi(r)|^2$, or equivalently the Fourier transform of the atomic form factor, and momentum-space distribution $n(k) = \int \frac{dr}{2\pi\hbar} W(k, r) = |\tilde{\Psi}(k)|^2$ with $\tilde{\Psi}(k) = \int \frac{dx}{2\pi\hbar} e^{-ikx/\hbar} \Psi(x)$. The $W(k, r)$ is an analogue of a Fourier transformed one-dimensional GPD. The impact parameter-dependent parton distributions [1], related to GPDs again by a Fourier transform with respect to the momentum transfer Δ_\perp , are transparently identified as relativistic nucleon Wigner distributions [2]. Thus, the studies of GPDs will shed the light on the phase-space distribution of quarks in the proton. GPDs are cleanly probed in deeply virtual Compton scattering involving only one hadron, the nucleon, whose structure is unraveled through electron scattering [3]. The same GPDs enter the amplitude of exclusive electroproduction of mesons in the asymptotic regime of large momentum transfer [4]. However due to the presence of an extra hadron in the final state and the specifics of perturbative QCD approach to such processes, one has to pose the question of applicability of hard gluon exchange mechanism at moderate photon virtualities. The cross section of meson photoproduction with longitudinally polarized γ^* is, see Fig. 1 (left),

$$\frac{d\sigma_L^M}{d|\Delta^2|d\varphi} = \frac{\alpha_{\text{em}}\pi}{\mathcal{Q}^6} \frac{f_M^2}{N_c^2} \frac{x_B^2}{(2-x_B)^2} \left\{ \sigma_M + \sigma_M^\perp \sin\Theta \sin(\Phi - \varphi) \right\}, \quad (1)$$

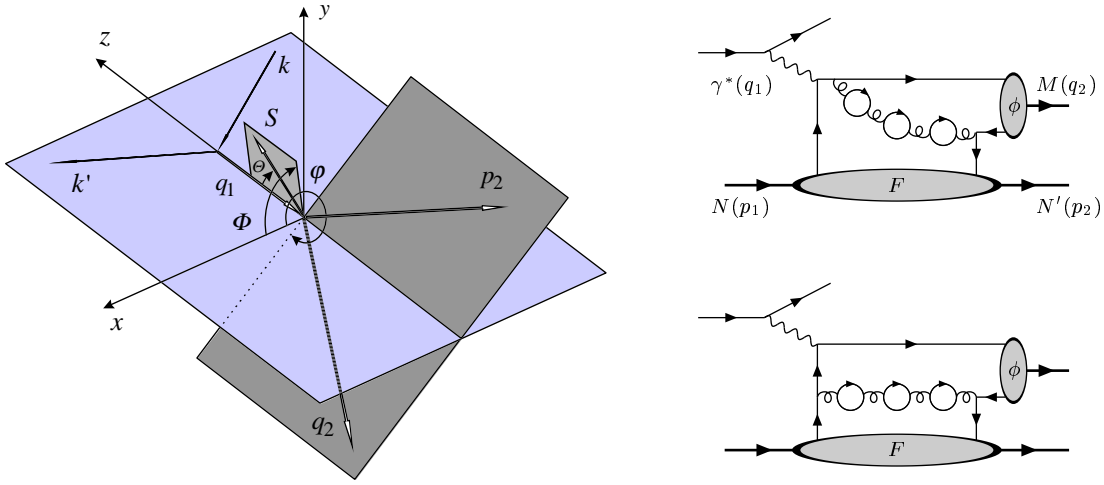


FIGURE 1. Kinematics of the exclusive meson electroproduction off the proton in its rest frame (left) and leading order perturbative diagrams in hard scattering approach (right) dressed by fermion bubble insertions which generate power corrections in the amplitude.

where $q_1^2 = -\mathcal{Q}^2$, $x_B = \mathcal{Q}^2/(2p_1 \cdot q_1)$, and M runs over mesons of different flavors. For the charged pseudoscalar meson $P = \pi^+$ the decay constant is $f_\pi = 132 \text{ MeV}$, while for the neutral vector mesons $V^0 = \rho^0, \omega$, they are $f_\rho = 153 \text{ MeV}$ and $f_\omega = 138 \text{ MeV}$. The parts of the cross sections read for target polarization-independent [5, 6, 7]

$$\sigma_P = 8(1 - x_B) |\widetilde{\mathcal{H}}_P|^2 - x_B^2 \frac{\Delta^2}{2M_N^2} |\widetilde{\mathcal{E}}_P|^2 - 4x_B^2 \Re \left(\widetilde{\mathcal{H}}_P^* \widetilde{\mathcal{E}}_P \right), \quad (2)$$

$$\sigma_V = 8(1 - x_B) |\mathcal{H}_V|^2 - x_B^2 \left(2 + (2 - x_B)^2 \frac{\Delta^2}{2M_N^2} \right) |\mathcal{E}_V|^2 - 4x_B^2 \Re (\mathcal{H}_V^* \mathcal{E}_V), \quad (3)$$

and target (transverse) polarization-dependent components

$$\sigma_P^\perp = -4x_B \sqrt{1 - x_B} \sqrt{-\frac{\Delta^2}{M_N^2}} \sqrt{1 - \frac{\Delta_{\min}^2}{\Delta^2}} \Im \left(\widetilde{\mathcal{H}}_P^* \widetilde{\mathcal{E}}_P \right), \quad (4)$$

$$\sigma_V^\perp = 4(2 - x_B) \sqrt{1 - x_B} \sqrt{-\frac{\Delta^2}{M_N^2}} \sqrt{1 - \frac{\Delta_{\min}^2}{\Delta^2}} \Im (\mathcal{H}_V^* \mathcal{E}_V), \quad (5)$$

respectively. The generalized structure function $\mathcal{F} = \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$ depends on the skewness $\xi = x_B/(2 - x_B)$, the t -channel momentum transfer Δ^2 and resolution scale \mathcal{Q}^2 . In leading-twist approximation, it is expressed as a convolution of the meson distribution amplitude $\phi(u)$, normalized to $\int_0^1 du \phi(u) = 1$, the quark or gluon GPD $F = \{H, E, \widetilde{H}, \widetilde{E}\}$ and, correspondingly, the quark or gluon coefficient function T via [4]

$$\begin{aligned} \mathcal{F}_M(\xi, \Delta^2; \mathcal{Q}^2) \\ \equiv \int_0^1 du \int_{-1}^1 dx \phi_M(u) \{ T_M(u, x, \xi; \mathcal{Q}^2) F_M(x, \xi, \Delta^2) + T_g(u, x, \xi) F_g(x, \xi, \Delta^2) \}. \end{aligned} \quad (6)$$

The function T_M to lowest order approximation is given by the one-gluon exchange mechanism displayed in Fig. 1 (right). The studies of higher-order perturbative corrections to the hard coefficient function in many physical observables have demonstrated that ambiguities generated by the perturbative resummation of fermion vacuum polarization insertions were of the same order of magnitude as available non-perturbative estimates of matrix elements of higher-twist operators. The development and sophistication of these ideas has led to some evidence that infrared renormalons might reflect the magnitude of higher-twist contributions and even their functional dependence on scaling variables and can thus be used as a rough estimate of power-suppressed effects. On the practical side to compute them in the present circumstances, one replaces the tree gluon propagator, in the single bubble-chain approximation, see Fig. 1 (right), by [in the Landau gauge]

$$\mathcal{D}_{\mu\nu}(k) = \frac{4\pi}{\alpha_s b} \int_0^\infty d\tau e^{-4\pi/(\alpha_s b)\tau} \left(\frac{\mu^2 e^C}{-k^2} \right)^\tau \frac{1}{k^2} \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right),$$

where $C_{\overline{\text{MS}}} = \frac{5}{3}$ in the $\overline{\text{MS}}$ and $C_{\text{MS}} = \frac{5}{3} - \gamma_E + \ln 4\pi$ in the MS scheme, and $b = \frac{11}{3}N_c - \frac{4}{3}T_F N_f$ is the first coefficient of the QCD beta-function and $\alpha_s = \alpha_s(\mu^2) = 4\pi/(b \ln \mu^2/\Lambda_{\overline{\text{MS}}}^2)$, where the last equality hold to one-loop order. The functions F_M which enter the above structure functions are combinations of q -flavor quark GPDs

$$F_\pi = F_u - F_d, \quad F_\rho = Q_u F_u - Q_d F_d, \quad F_\omega = Q_u F_u + Q_d F_d, \quad (7)$$

where the quark charges are $Q_u = \frac{2}{3}$ and $Q_d = -\frac{1}{3}$. For π^+ and V^0 only the polarized $F = \{\tilde{H}, \tilde{E}\}$ and, correspondingly, unpolarized GPDs $F = \{H, E\}$ enter the game. The quark coefficient function (with resummed renormalon chains) for the $M = \pi^+$ has the form

$$T_\pi(u, x, \xi; \mathcal{Q}^2) = \frac{4\pi C_F}{b} \int_0^\infty \frac{d\tau}{\xi} e^{-4\pi/(\alpha_s b)\tau} \left(\frac{2\mu^2 e^C}{\mathcal{Q}^2} \right)^\tau \times \left\{ \frac{Q_u}{[\bar{u}(1 - \frac{x}{\xi} - i0)]^{\tau+1}} - \frac{Q_d}{[u(1 + \frac{x}{\xi} - i0)]^{\tau+1}} \right\}, \quad (8)$$

with $C_F = (N_c^2 - 1)/2N_c$. The coefficient function for neutral vector mesons $M = V^0$, T_V , is obtained from this one by setting $Q_u, Q_d \rightarrow 1$ since the quark charges are included into flavor combinations of GPDs. Finally, for completeness we present the leading-order gluon coefficient function contributing to neutral vector meson production,

$$T_g(u, x, \xi) = \frac{\alpha_s}{\xi^2} \frac{4T_F \sum_q Q_q}{u\bar{u}(1 - \frac{x}{\xi} - i0)(1 + \frac{x}{\xi} - i0)}.$$

If one absorbs the dependence on the momentum fraction into the argument of the coupling, $\alpha_s(\frac{1}{2}u(1 \pm \frac{x}{\xi})\mathcal{Q}^2 e^{-C})$, one explicitly sees that the end-point regions produce divergences. Infrared renormalons are caused by the end-point singularities [Feynman

mechanism] in exclusive amplitudes [8], see also [9, 10]. This can be viewed as an estimate of the ambiguity in the resummation of higher-order perturbative corrections or, taken to the extreme, as a model of higher-twist contributions [11]. Convolution of the coefficient function with the distribution amplitude generates renormalon poles. For the asymptotic distribution amplitude $\phi_{\text{asy}}(u) = 6u\bar{u}$ one gets two poles $\tau = 1$ and $\tau = 2$, corresponding to ambiguities on the level of \mathcal{Q}^{-2} and \mathcal{Q}^{-4} power corrections. Since the latter receives extra contributions from higher order diagrams as well, we use only $\tau = 1$ pole for the estimates of the form of higher-twist corrections. Taking the imaginary part (divided by π) arising from the contour deformation around the renormalon poles as a measure of their magnitude, we get

$$\widetilde{\mathcal{H}}_\pi(\xi, \Delta^2; \mathcal{Q}^2) = \widetilde{\mathcal{H}}_\pi^{\text{PV}}(\xi, \Delta^2; \mathcal{Q}^2) + \theta \frac{\Lambda_{\overline{\text{MS}}}^2 e^{5/3}}{\mathcal{Q}^2} \int_{-1}^1 dx \Delta_{\tilde{H}}(x, \xi) \tilde{H}_\pi(x, \xi, \Delta^2), \quad (9)$$

where $\theta = \pm 1$ comes from the ambiguity to go around the renormalon pole in the Borel plane. Here we have used the one-loop expression for the QCD coupling constant and

$$\Delta_{\tilde{H}}(x, \xi) = 48 \frac{\pi C_F}{b\xi} \left\{ \frac{Q_u}{(1 - \frac{x}{\xi} - i0)^2} - \frac{Q_d}{(1 + \frac{x}{\xi} - i0)^2} \right\}, \quad (10)$$

Since the GPD and its first derivative are continuous functions at $x = \pm \xi$ [12], the second integral is well-defined. In the first term of (9) one uses the principal value prescription to go around the poles in the Borel plane. For the \tilde{E} we use the pion-pole dominated form [7] and get

$$\tilde{\mathcal{E}}_\pi(\xi, \Delta^2; \mathcal{Q}^2) = \tilde{\mathcal{E}}_\pi^{\text{PV}}(\xi, \Delta^2; \mathcal{Q}^2) - \theta \frac{\Lambda_{\overline{\text{MS}}}^2 e^{5/3}}{\mathcal{Q}^2} \Delta_{\tilde{E}}(\xi, \Delta^2; \mathcal{Q}^2), \quad (11)$$

where we have kept only the single and double poles at $\tau = 1$ in the second term, so that

$$\Delta_{\tilde{E}}(\xi, \Delta^2; \mathcal{Q}^2) = 72 \frac{\pi C_F}{b\xi} F_\pi(\Delta^2) \left(2 + \ln \frac{\Lambda_{\overline{\text{MS}}}^2 e^{5/3}}{\mathcal{Q}^2} \right). \quad (12)$$

In the vicinity of the pion pole one can approximate $F_\pi(\Delta^2) = 4g_A M_N / (m_\pi^2 - \Delta^2)$.

In estimates, shown in Fig. 2, we relied on GPDs deduced from an ansatz based on modeling the double distribution as a product [12] $\Delta F(y, z, \Delta^2) = \pi(y, |z|) \Delta f(y, \Delta^2)$ of exclusive profile π and an inclusive parton distribution augmented to have an intrinsic momentum-transfer dependence $\Delta f_q(y, \Delta^2) = \eta_q A_q x^{a_q - \alpha'_q \Delta^2 (1-x)} (1-x)^{b_q} (1 + \gamma_q x + \rho_q \sqrt{x})$ with parameters fixed by the GSA forward densities [13] in $\Delta^2 = 0$ limit and slopes $\alpha'_u = 1.15 \text{ GeV}^{-2}$, $\alpha'_d = 1.0 \text{ GeV}^{-2}$ chosen to fit the dipole form of the axial form factor with the effective mass $m_A^2 = 0.9 \text{ GeV}^2$. We give the cross section [at leading order compatible with earlier estimates [15, 6]] and transverse target-spin asymmetry $\mathcal{A}_P^\perp = (2\sigma_P)/(\pi\sigma_P^\perp)$. In our evaluations we set $\theta = 1$ and $\Lambda_{\overline{\text{MS}}} = 280 \text{ MeV}$ for $N_f = 4$ and use the tree level result for $\mathcal{F}^{\text{PV}} \rightarrow \mathcal{F}^{\text{LO}}$. Note however that in calculations of higher-twist corrections via renormalons in deeply inelastic scattering in order to get the right

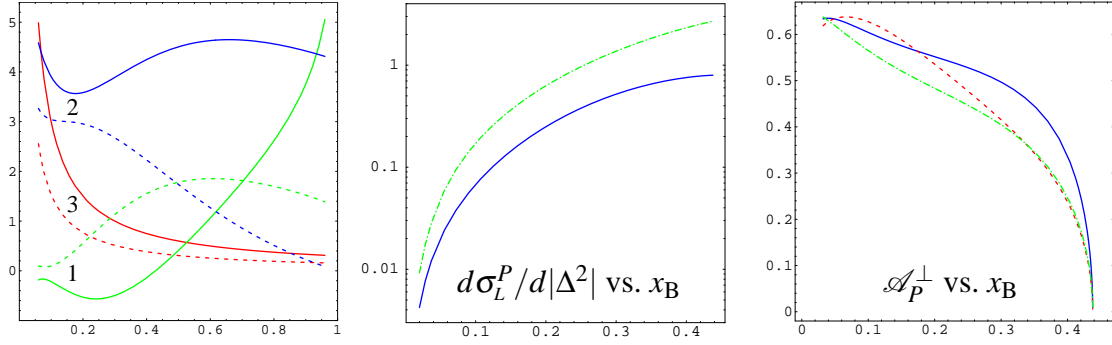


FIGURE 2. Generalized structure functions (left) in leading twist approximation (dashed) and including twist-four corrections (solid) as a function of x_B for $\Delta^2 = -0.3 \text{ GeV}^2$ and $\mathcal{Q}^2 = 10 \text{ GeV}^2$: (1) $\text{Re} \widetilde{\mathcal{H}}$, (2) $\text{Im} \widetilde{\mathcal{H}}$, and (3) $10^{-2} \cdot \widetilde{\mathcal{E}}$. The photoproduction cross section in units of nbarns (middle) without (solid) and with (dash-dotted) power suppressed contributions for the same values of the kinematical variables. The transverse spin asymmetry (right) at leading order (solid) and with twist-four power effects taken into account for $\Delta^2 = -0.3 \text{ GeV}^2$ and $\mathcal{Q}^2 = 4 \text{ GeV}^2$ (dashed) and $\mathcal{Q}^2 = 10 \text{ GeV}^2$ (dash-dotted). The maximal value of $x_{B,\text{max}}$ is set by the kinematical constraint $|\Delta^2| > |\Delta_{\text{min}}^2| = M_N^2 x_B^2 / (1 - x_B)$.

magnitude of experimental data one has to take a larger value $|\theta| \approx 2 - 3$ [14]. The extremely large power corrections to the absolute cross section of pion leptonproduction are in qualitative agreement with the earlier consideration in Ref. [16]. As we observe, however, the renormalon model of higher-twist contributions affects in a marginal way the asymmetry and thus leads to the apparent conclusion of *the precocious scaling* in ratios of observables, — a fact pointed out previously in various circumstances [5, 6, 17].

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